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Students' construction error in translation among mathematical representations

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Abstract. This research arises from student failure to construct target representation equivalent to source representation. The purpose of this research was to investigate student construction errors in translation among mathematical representations. Explorative study with a qualitative approach was conducted to 51 students majoring in mathematics. The data collection tools used were the mathematical translation task and interview protocol. Data were collected by think-aloud and task-based interviews. The results showed that student construction errors in translation among mathematical representations occur in various forms, i.e. lost of representational attributes, interference thinking, pseudo-thinking, disconnected connections, implementation error and lack of preservation of representative equivalence. The this research findings have implications for designing scaffolding of learning-oriented reconstruction for students' thinking to make meaningful translations.

1. Introduction

Translations among mathematical representations have a strong influence on mathematical ability, such as concept comprehension and mathematical problem-solving. The inability to translate causes failure in mathematical problem-solving [1]. In addition, the lack of coordination among different representational structures in translations causes to a compartmentalized understanding [2]. Therefore, the theme of the students' errors in translation is still the center of attention in today's mathematics education research.

The translation errors among mathematical representations have been taken from various perspectives that are text-based [3], representation-based [4], and translation process-based [5]. From the perspective of the translation process, defined three types of translation errors: misinterpretation, implementation errors, and preservation errors. This condition shows that students have difficulty in almost every translation process [5]. To be able to overcome these difficulties can be started by conducting studies on student construction activities during translation.

In the context of translation, construction activity is seen as an action, technique or heuristic [6,7]. Moreover, the construction activity in this research refers to how students in evoking and providing meaning towards source representations attributes, establish connections among source representations' attributes and targets, building targets, and ensure representational equivalence of the source and target representations. Specifically, this research aims to investigate student construction errors in translation among mathematical representations. Construction errors are investigated from each of the translational stages i.e. unpacking the source, preliminary coordination, constructing the

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target and determining equivalence [8]. This construction error investigate becomes important to do because it can help teachers in preparing to scaffold in order to reconstruct students' thinking in order to make meaningful translations.

2. Method

The qualitative explorative method was applied in this research. The purposive sampling method is used to select the research subject. Fifty-one students department of the mathematics Universitas Negeri Malang, Indonesia were chosen based on the consideration that they had studied calculus. The supporting instruments for data collection were the mathematical translation task (Figure 1) and task-based interviewing protocols.

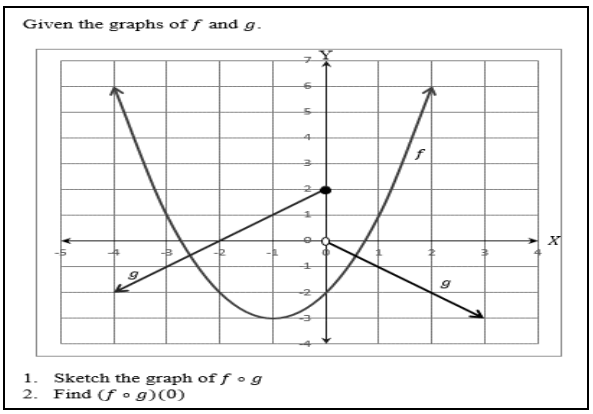


Figure 1. The mathematical translation task

In Figure 1 there are graphs of f and g, students are asked to sketch a composition function of f with f and determine $f \circ g(0)$. The mathematical translation task (MTT) had been validated by mathematicians and mathematics education and it was valid to allow students to translation among mathematical representations. The task-based interview protocol contained open-ended questions to clarify students' thinking processes during MTT solving. Data collection was conducted in the odd semester of academic year 2017/2018. Data collection was done in two stages: to ask students to think aloud in MTT solving and conducting the interview. Data were analyzed by reduced, categorized or coded and described the forms of construction errors in translation among mathematical representations.

3. Result and Discussion

In the mathematical translation task (MTT), students were asked to draw a graph of the composition function f with g $(f \circ g)$ of the graph of f and g given. The strategy of the majority (96%) of students were to translate from graphs of f and g to their symbolic form, to determine the symbolic of $f \circ g$ and translation from symbolic of $f \circ g$ to graph of $f \circ g$. Referring to the work of students, about 53% of students were wrong or failed to sketch an graph of $f \circ g$.

The location of the students' error varied thirty-three percent of students were mistaken on translation from graph of f to symbolic of f, 17% of students were mistaken on translation from graph of g to symbolic of g, 11% of students wrong determine symbolic of $f \circ g$, and 39% of students were wrong on the translations from symbolic of $f \circ g$ to graph of $f \circ g$. This finding was in contrast to previous research [7] that translation from symbolic to graph was quite easy for students compared to translation from graph to symbolic. The explanation of why many students were failed on translation from graph to symbolic could be known in the analysis of construction errors at the stage of the unpacking the source. Analysis of construction students' error are described based on each stage of the translation process as follows.

3.1. Construction Error on Unpacking the Source

Unpacking the source representation was the first step on translation among mathematical representations. At this stage, there were three activities that students needed to do, which was to find the source representation attribute, to separate the relevant attributes to the irrelevant and to investigate the properties of the source representation to predict the attributes of the target representation [8]. The purpose of the three activities at this stage was to construct the source representation attribute by specifying the source representation attribute to be encoded/mapped to the target representation attribute. This attribute construction error caused the student to be failed in building the target representation. In this research, attributes construction error occurred when students did the translation from symbolic of $f \circ g$ to graph of $f \circ g$.

In the translation from symbolic of $f \circ g$ to graph of $f \circ g$, which was acting as the source representation, was $f \circ g = \begin{cases} (x+3)^2 - 3, & x \leq 0 \\ (-x+1)^2 - 3, & x > 0 \end{cases}$. Many students could not identify key attributes of quadratic functions in symbolic form. As seen in the example of student work (Figure 2). The strategy adopted by students in constructing the graph of $f \circ g$ is by substituting some values of x to the symbolic of $f \circ g$, so that some paired pairs were obtained $(x, f \circ g(x))$ presented in tabular form. Then, the students looked at the ordered pair as the points to be plotted on the Cartesian coordinates. The strategy pursued by the students was procedural.

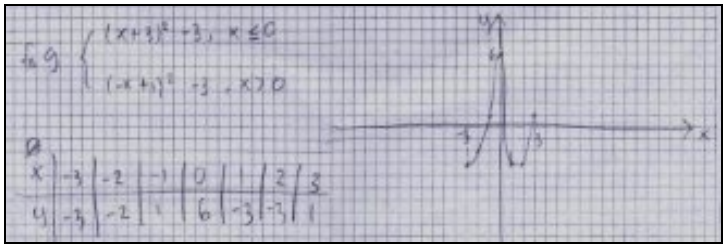


Figure 2. The work of students unpacking the source

From Figure 2, it is seen that students could not see the ordered pairs $(-3, -3)$ and $(1, -3)$ as symbolic key attributes. Whereas the general form of source representation was $f \circ g = \left(x - (-h)\right)^2 + (-k)$, where (h, k) was the peak of the parabola. It meant that the source representation had no the fact gaps and the interpretations used in unpacking attributes were classified as local interpretations [5]. These two nature were the reasons why symbolic translation to graphs was easy [7].

Contrary to previous research [7], this research revealed the fact that translation from symbolic to graphs was difficult for students. The root of the problem was that students could not find the key attribute of representation that was clearly available explicitly. By and large, the students experienced an error in constructing the source representation attribute, which was missing the attribute of the source representation.

3.2. Construction Error on Preliminary Coordination Stage

In the preliminary coordination stage, students mapped the source representation attribute into the target representation attribute [8]. In mapping stage, students would construct an idea network that described the connection between the source representation attributes and the target representation attributes. Based on data analysis result obtained three forms of student error to construct network between the attribute of the source and target representation. Description of each error, described in the following paragraphs.

The first form construction error occurred when the student mapped the graph attribute of f in the graphic character of f of the graph $y = x^2$ which was moved one unit to the left and descended three units into the symbolic attribute of $f(x) = (x-3)^2 + 1$. This mapping activity could be seen from the works of the students available in Figure 3.

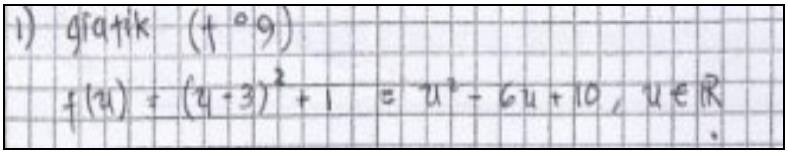


Figure 3. Student results in mapping the graph attribute f to symbolic attribute f

From figure 3, it could be seen that the mapping of representational attributes which was conducted by students was classified as the conceptual one because the mapping was conducted through interpreting attribute characters of the graph representation [7]. Nevertheless, the students were mistaken in choosing symbolic representation attributes which were suitable for the nature of the graph f. From previous learning experience, the student already had a scheme of graphic characteristics of $y = (x - a)^2$ which was the graph of $y = x^2$ which is shifted by a unit to the left and graphic characteristics of $y - b = x^2$ which was the graph of $y = x^2$ which shifted by b unit down. However, when the students were conducting preliminary coordination, students experienced obstacles in recalling the schemes that had been constructed. These two schemes appeared simultaneously in the students' minds so that these two schemes were mixed, thereby impeding the preliminary coordination. Referring to the theory of information processing [9], this construction error is called interference thinking.

The second construction error occurred when the student mapped the graph attribute of g to the symbolic attribute of g. The student had set three points on the graph of g as an attribute to be mapped to the general form of the function. In the student work (Figure 4) it was seen that the students made a relationship between three points on the graph of g and the function attribute of $g(x) = ax^2 + bx + c$ i.e. coefficients x^2 (a), coefficients x (b) and constants (c). Then the students substituted the three points on the graph of g to $g(x) = ax^2 + bx + c$, thus it obtained $a = 0$, $b = 1$ and $c = 2$.

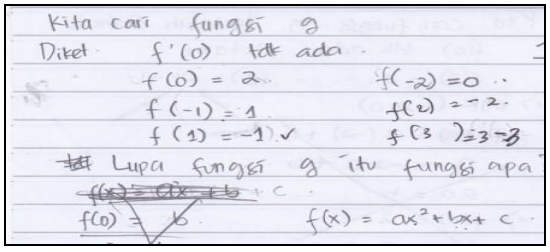


Figure 4. The works of students in mapping attribute graph of g to atribute symbolic of g.

From Figure 4 it is seen that the symbolic form of the resulting graph g is correct, but the student's thinking process is "concept flawed". This showed that the thinking process of students in the preliminary coordination was considered as a pseudo-thinking [10–12]. Students experiencing pseudo-thinking in MTT solving could produce correct target representation, however, it was not obtained from the correct construction process [10].

The third form construction error occurred when the students were mapping the symbolic attribute of $f \circ g$ to the graph of $f \circ g$. Students realized, that to draw a graph of the a quadratic function, the direction of the opening of a parabola is needed, the intersection of the x-axis and the y-axis. However, students did not find the attributes of $f \circ g = \begin{cases} x^2 + 6x + 6, & -\infty < x \leq 0 \\ x^2 - 2x - 2, & 0 \leq x < 0 \end{cases}$ which could be mapped to the graph attribute quadratic function. This could be seen from the works of students in Figure 5.

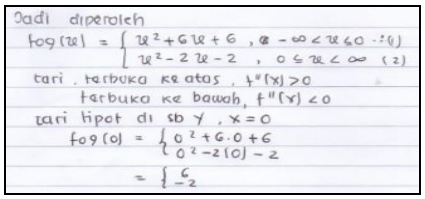


Figure 5. The students' works in mapping attribute symbolic of $f \circ g$ to atribute graph of $f \circ g$

Figure 5 showed that the student did not connect the coefficients of x^2 with the direction of the parabolic opening. In addition, the students did not relate the constant value to the quadratic function as the y value at the graph's point of intersection with the y-axis. This indicated that students were unable to establish connections between source and target representation structures [13]. This student error was called as disconnected connection in the preliminary coordination stage

3.3. Construction Error in Target Contructing Stage

A common error in target constructing is that the student is not correct in executing the target representation attributes that have been obtained from the previous translational process. This error repeatedly came up when students were drawing a graph of $f \circ g$ from the symbolic of $f \circ g$.

The student already had the point $(-3, -3)$ as the peak point, $(0,6)$ as the y-axis intersection and $(-3+\sqrt{3}, 0), (-3-\sqrt{3}, 0)$as the intersection of x-axis. However, when the students plotted the dots, they made a error which caused irrelevant or wrong graphic outcomes. Graph of $f \circ g$ made by the students could be seen at Figure 6.

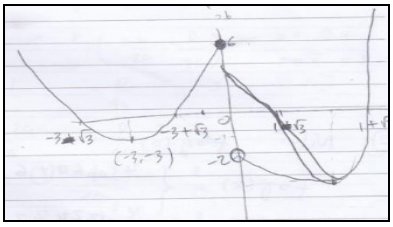


Figure 6. Students' works in sketching $f \circ g$

In Figure 6 students are mistaken in plotting the point (-3, -3) in Cartesian coordinates. Referring to the results of the research [5], the error is classified as an implementation error.

3.4. Construction Error in Determining Equivalence Stage

One of the factors that caused students were not able to construct a suitable target representation to the source representation was the lack of preservation of representative equivalence of both representations. Meanwhile, ensuring the equivalency between the source and target representation was the ultimate goal of translation [14]. From the results of interviews with students who experienced errors in constructing the target, it was revealed that they did not check on the suitability of target representation with the source representation. Here is the excerpt from the interview.

R : *What makes you believe that the graph of $f \circ g$ already represents the symbolic of $f \circ g$?*
S : *Because the graph that I created was parabolic.*
R : *Did you check the suitability between the graph of $f \circ g$ and symbolic of $f \circ g$?*
S : *I did not do it.*

Based on the interview results a fact was revealed that the students did do not have an attempt to ensure that the graph of $f \circ g$ had already been drawn really represented the symbolic of $f \circ g$. Whereas by taking sketch of the source representation from the point of view of the target

representation attribute would make the student to avoid the construction error in translation [5]. It was predicted that the learning experience, teacher disposition and, student beliefs caused students could not be accustomed to determining representative equivalencies of the source and target representation. Therefore, research needed to be done to test these allegations.

4. Conclusion

This research shows that students have difficulty in translating from source to target representation. Most students experience construction errors at every stage of translation. The forms of student construction errors in translation among mathematical representations are a loss of representation attributes, interference thinking, pseudo-thinking, disconnected connections, implementation errors and lack of preservation of representative equivalence. Based on these findings, it is assumed that mathematical learning has not taught how to reason in translation among mathematical representations. Therefore, the need for innovation in learning so that students in doing translation based on logical reasoning. It can be started by reconstructing students' thinking process through developing scaffolding in mathematics learning. In addition, it needs to do a search related to the kind of knowledge that students and teachers need to have in order to make meaningful translations. This theme is proposed for further research.

Acknowledgments

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References

[1] Pape S J 2004 *J Res Math Educ* **35** 187-219
[2] Gagatsis A, Elia I, Mousoulides N 2006 *Relime Número Espec* **9** 197-224
[3] Mangulabnan P A T M 2013 *US-China Educ Rev A* **3** 365-73
[4] Adu-Gyamfi K, Bossé M J, Chandler K 2015 *Int J Math Teach Learn* **1** 1-29
[5] Adu-Gyamfi K, Stiff L, Bossé M J 2012 *School Science and Mathematics* **112** 159-70
[6] Leinhardt G, Zaslavsky O, Stein M 1990 *American Educational Research Association* **60** 1-64
[7] Bossé M J, Adu-Gyamfi K, Cheetam M R 2011 *Int Electron J Math Educ* **6** 113-33
[8] Bossé M J, Adu-Gyamfi K, Chandler K 2014 *Int J Math Teach Learn* **28** 1-28
[9] Stemberg R J, Stemberg K 2012 *Cognitive Psychology Sixth Edition* (USA: Wadsworth, Cengage Learning)
[10] Subanji S, Maedi S 2015 *Journal oh The Korean Society of Math Educ Series D : Research in Math Educ* **19** 55-73
[11] Rofiki I, Nusantara T, Subanji S, Chandra T 2017 *J of Phys: Conference Series* **943** 1-8
[12] Vinner S 1997 *Educ Stud Math* **34** 97-129
[13] Adu-Gyamfi K, Bossé M J, Chandler K 2016 *Int J Sci Math Educ* **15** 915-38
[14] Bossé M J, Adu-Gyamfi K A, Cheetam M R 2011 *Int Electron J Math Educ* **6** 1-23